



RAN-2303080101040004

M. Sc. Physics (Sem. - I) Examination November - 2025

PH - 414 Elective 3: Numerical Methods

[Total Marks: 70

સૂચના : / Instructions

- (1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book
- (2) Symbols used have their usual meaning.
- (3) Figures to the right indicate marks.

Seat No.:

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Student's Signature

Q. 1 Attempt any two questions out of the (a), (b) and (c) below. 18

- (a) (i) The function $y = x^2 + 9$ is tabulated as (3, 36), (4, 73), (5, 134). Predict the value of y (4.5) using quadratic and natural cubic splines and state the absolute error in each case. 05
- (ii) Evaluate the Recurrence relation of quadratic spline. 04
- (b) (i) Given the set of data points (1,1) (2, 5), (3,11) and (4,8) satisfying the function $y = f(x)$, Find the Linear and cubic splines satisfying the given data. Determine the approximate values y (1.5). 05
- (ii) Discuss the minimizing property of cubic Spline. 04
- (c) (i) For the knots 0, 1,2, 3, 4, compute the cubic B splines at $x = 1$ and $x = 2$ using the formula of the Cox and de Boor. 05
- (ii) Define the natural cubic spline and state the end conditions for D1 and D2 spline. 04

Q. 2 Attempt any two questions out of the (a), (b) and (c) below. 18

- (a) (i) Find the solution of following tridiagonal system. 05

$$\begin{aligned} 2x - y &= 50 \\ -x + 2y - z &= 1 \\ -y + 2z &= 1 \end{aligned}$$
- (ii) What is pivot number? Write the steps to follow in Gauss Elimination method and importance. 04
- (b) (i) Determine the eigenvalues and corresponding eigenvectors of the following matrix. 05

$$\begin{bmatrix} -2 & 1 & 1 \\ -11 & 4 & 5 \\ -1 & 1 & 0 \end{bmatrix}$$
- (ii) Given the matrix 04

$$\begin{bmatrix} 1 & 3 & 5 \\ 1 & 4 & 3 \\ 1 & 3 & 2 \end{bmatrix}$$

Find $\|A\|_1, \|A\|_e$, and $\|A\|_\infty$...

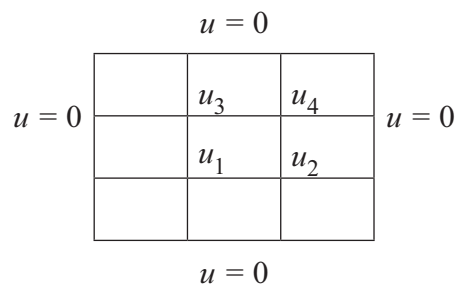
- (c) (i) Solve the following system by the Jacobi method. 05
 $6x + y + z = 20$
 $x + 4y - z = 6$
 $x - y + 5z = 7$
- (ii) What is vector norms? Write the rules that vector norms follow. Also, discuss the types of vector norms. 04

Q. 3 Attempt any two questions out of the (a), (b) and (c) below. 18

- (a) (i) Find Taylor's series solution of equation $\frac{dy}{dx} = x - y^2$ with $y(0) = 1$. 05
- (ii) Using Adam-Moulton method, find $y(1.4)$. Given $\frac{dy}{dx} = x^2(1 + y)$ with $y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979$. 04
- (b) (i) Given $\frac{dy}{dx} = x + y^2$ with $y(0) = 1$, find $y(0.2)$ where $h = 0.1$ using RK method of 4th order. 05
- (ii) Given $\frac{dy}{dx} = x^2 + y$ with $y(0) = 1$, find $y(0.02)$ & $y(0.04)$ by Euler's modified method. 04
- (c) (i) Using the Picard's method of successive approximations, find the third approximation of the solution of the equation: $\frac{dy}{dx} = x + y^2$ Where $y = 0$ when $x = 0$. Find the solution for $y(1)$ and $y(2)$. 05
- (ii) Discuss the Picard's method of successive approximation. 04

Q. 4 Attempt any two questions out of the (a), (b) and (c) below. 16

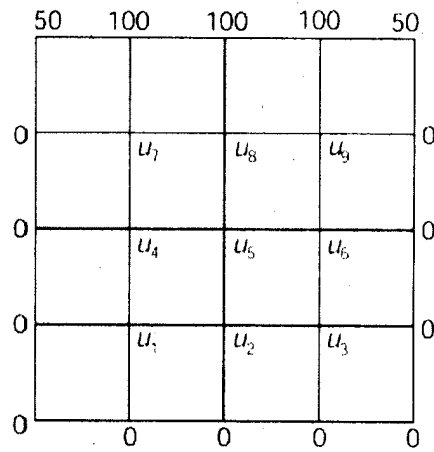
- (a) (i) Solve the Poisson's equation $\nabla^2 u = 8x^2y^2$ for the square grid shown below ($h = 1$). 05



- (ii) Deduce the standard five-point formula. 03
- (b) (i) Solve the heat conduction problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the condition $u(x, 0) = u(0, t) = 0$ and $u(1, t) = t$. Use Crank Nicolson formula to compute the value of $u\left(\frac{1}{2}, \frac{1}{16}\right)$. Take $h = \frac{1}{2}$ and $l = \frac{1}{16}$ 05
- (ii) Discuss the three-level difference scheme. 03

(c) (i) Solve Laplace's equation for the figure given below.

05



(ii) Derive the Bender-Schmidt recurrence formula.

03
